Chapter 2 ELECTROMAGNETIC BASICS

A The Electromagnetic Spectrum - EM Wave theory

An understanding of the electromagnetic spectrum, and electromagnetic (EM) radiation is the first essential item we must address. Light, radar waves, radio waves, and all the rest are at their core, electromagnetic radiation. We briefly describe the underlying physical equations, and the nature of the wave equation that results from them.

The fundamental principles that define electricity and magnetism were codified by Maxwell in the late 1800's in four equations that bear his name:

1) $\oiint \vec{\mathbf{E}} \bullet d\vec{\mathbf{S}} = \dfrac{Q}{e_o}$ or $\nabla \bullet \vec{\mathbf{E}} = \dfrac{r}{e_o}$ (Eqn. 2.1 a)

2) $\oiint \vec{\mathbf{B}} \bullet d\vec{\mathbf{S}} = 0$ or $\nabla \bullet \vec{\mathbf{B}} = 0$ (Eqn. 2.1 b)

3) $\oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{l}} = -\dfrac{\partial}{\partial t} \iint \vec{\mathbf{B}} \bullet d\vec{\mathbf{S}}$ or $\nabla \times \vec{\mathbf{E}} = -\dfrac{\partial \vec{\mathbf{B}}}{\partial t}$ (Eqn. 2.1 c)

4) $\oint \vec{\mathbf{B}} \bullet d\vec{\mathbf{l}} = m_o \, \mathbf{i} + m_o e_o \dfrac{\partial}{\partial t} \iint \vec{\mathbf{E}} \bullet d\vec{\mathbf{S}}$ or (Eqn. 2.1 d)

$$\nabla \times \vec{\mathbf{B}} = m_o \, \mathbf{J} + m_o e_o \frac{\partial \vec{\mathbf{E}}}{\partial t}$$

These four equations respectively say:

1) that the electric flux through a "Gaussian" surface is equal to the charge contained inside the surface;
2) that in the absence of a magnetic point charge (we don't know of any), the magnetic flux through a Gaussian surface is equal to zero;
3) That the voltage induced in a wire loop is defined by the rate of change of the magnetic flux through that loop (the equation that defines electrical generators); and
4) That a magnetic field is generated by a current (typically in a wire), but also by a time varying electric field.

These equations can be manipulated in differential form to produce a new differential equation, the wave equation:

$$\nabla^2 \vec{\mathbf{E}} - e_o m_o \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2} = 0 \quad \text{and} \quad \nabla^2 \vec{\mathbf{B}} - e_o m_o \frac{\partial^2 \vec{\mathbf{B}}}{\partial t^2} = 0 \qquad \text{(Eqn. 2.2)}$$

Maxwell understood that the solutions to these equations were defined by oscillating electric and magnetic fields ($\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$). In particular, these equations immediately give the speed of light: $c = \dfrac{1}{\sqrt{e_o m_o}}$. The complexity of the solutions vary, but there are some fairly simple ones involving plane waves, propagating in a straight line. Like all wave phenomena, the solution involves the wavelength, frequency, and velocity of the radiation. In equation form, a typical solution looks like this:

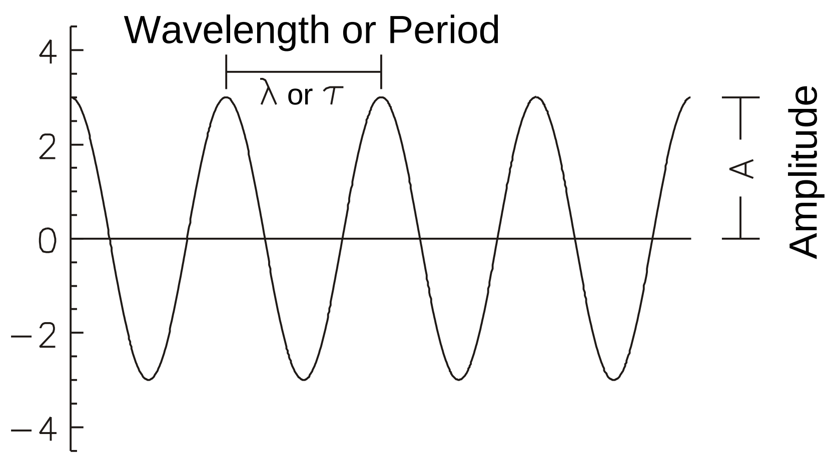
$$\vec{\mathbf{E}}(z,t) = E\hat{\mathbf{x}}\cos 2p\left(\frac{z}{l} - f\,t\right) \text{ or } \vec{\mathbf{E}}(z,t) = E\hat{\mathbf{x}}\cos(kz - wt) \quad \text{(Eqn. 2.3)}$$

which is the equation for a wave propagating in the plus $\hat{\mathbf{z}}$ direction, with an electric field polarized in the $\hat{\mathbf{x}}$ direction. (More on polarization below.)

The various terms in this equation are defined as follows:

E	= Amplitude of the electric field
l	= Wavelength (in meters)
k	= Wavenumber (meters^{-1})
f	= Frequency in Hz (cycles/sec)
ω	= Angular Frequency (radians/sec)
c	= Phase velocity of the wave (in m/sec)

Figure 2-1. Four cycles of a wave are shown, with wavelength λ, or period τ. The wave has an amplitude, A, equal to 3.



The solution depends upon the wavelength and frequency, which are related by:

$$l\,f = c \quad \text{(Eqn. 2.4)}$$

Note that the period, τ, is the inverse of the frequency, since for a wave it must be true that: $f \cdot \tau = 2\pi$. One can also define the angular frequency: $w = 2p\,f$, and the wavenumber: $k = 2p/l$.

For vacuum the value of c = 2.998 x 10^8 (m/s), an important constant of physics. The range of wavelengths, which will be of interest to us, covers a very large range of about 20 orders of magnitude. This range is illustrated in Figure 2-2.

This construct was the initial one for light and other forms of EM radiation, but at the turn of the 20th century, it became obvious that a different perspective was needed to explain some of the interactions of light and matter - in particular processes such as the photo-electric effect (and similar processes which are important for the detection of light). This led to a resurgence of the idea that light, or electromagnetic radiation, might better be thought of as a particle, dubbed the photon. The energy of an individual photon is given by

$$E = hf \quad \text{(Joules or eV)} \qquad \text{(Eqn. 2.5)}$$

where $\quad f$ = frequency of the EM wave (in Hz) and

$$h = \text{Planck's Constant} = \begin{Bmatrix} 6.626 \times 10^{-34} \text{ Joule-seconds} \\ 4.136 \times 10^{-15} \text{ eV-seconds} \end{Bmatrix}$$

The electron-volt (eV) is a convenient unit of energy and is related to the usual unit (Joule) by:
1 eV = 1.602 x 10^{-19} Joules

The photon energy, E, is determined by the frequency of the electromagnetic radiation. The higher the frequency, the higher the energy. Photons move at the speed of light, as expected for electromagnetic radiation. They have zero rest mass, however, so the rules of special relativity are not violated.

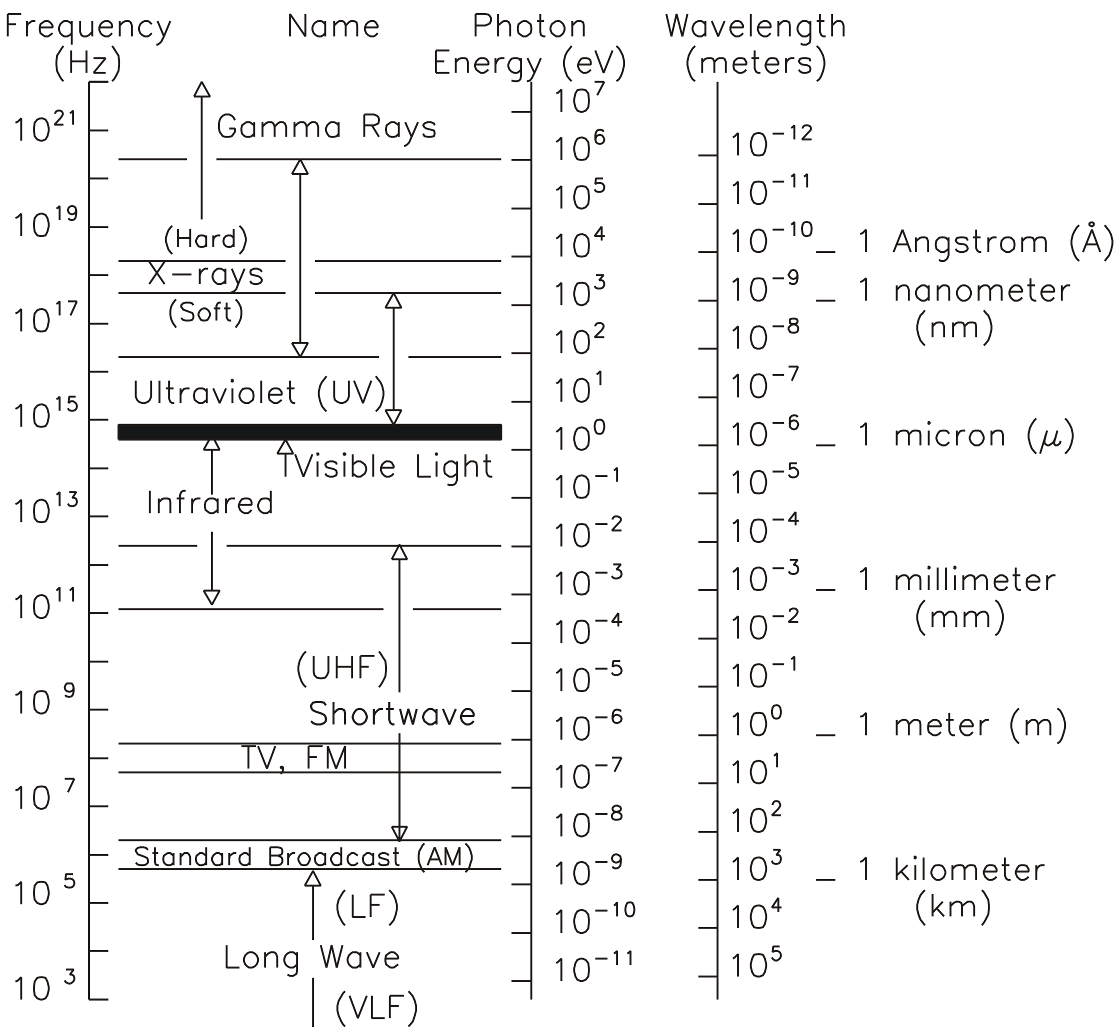


Figure 2.2 The spectrum of electromagnetic radiation.

Example:

The energy of the photons making up an electromagnetic wave (light wave) in the visible part of the spectrum (green color) is

$$E = hf = (4.14 \times 10^{-15})(6 \times 10^{+14}) = 2.48 \text{ eV}$$

which is on the order of (or slightly less than) typical atomic binding energies.

Energies of typical x-ray photons are in the 10^4 to 10^5 eV range, while the photons of a 100 MHz radio signal are only about 4×10^{-7} eV.

This business of energy in photons shows up as being very important for detector technology. One illustration of this is in the photoelectric effect. This is the one for which Albert Einstein won the Nobel Prize. The process that can be observed is illustrated here. If you shine light on a metal surface in a vacuum, electrons are liberated from the surface, and can be collected on a second surface, here, the collector plate. This process results in a flow of current that depends on the intensity of the light.

The energy of the electrons can be measured by applying a back bias (a negative voltage) to the collector plate, repelling the electrons. Voltages of a volt or so are typically sufficient to zero out the current. Traditional wave theory predicts that the amplitude of the current would vary with the amplitude of the light, which it does, but does not provide a way to explain an observed dependence on the wavelength (frequency) of the light. The higher the frequency of the light (the bluer the light), the more energy the electrons have. Einstein combined the above concept, E = hf, with the idea of a "work function", a fixed amount of energy necessary to liberate electrons from a metal surface - typically a volt or 2. In short, the light must possess enough energy to overcome the work function. If the wavelength is too large (the frequency too low), then no electrons are emitted from the surface.

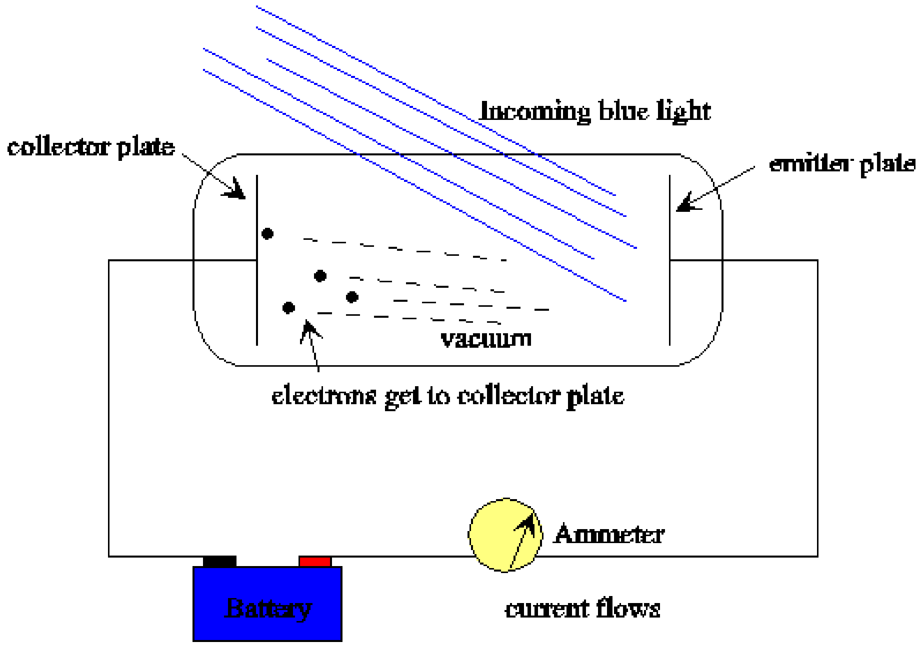


Figure 2.3 Layout for demonstration of the photoelectric effect.

In our illustration here, light at three wavelengths is used:
λ= 435.8 nm, 546.1 nm, and 632.8 nm. (blue, green, and red).
Calculating the energies corresponding to these wavelengths, we get:

$$E = hf = \frac{hc}{l} = \frac{4.136 \times 10^{-15} \text{ eV-s} \bullet 3\times10^{8} \frac{m}{s}}{435.8 \times 10^{-9} \text{ m}} = \frac{1.24\times10^{-6}}{4.358 \times 10^{-7}} = 2.85 \text{ eV}$$

Similarly, we get E=2.27 eV and E = 1.96 eV for 546.1 and 632.8 nm respectively. The experimental data shown below show that if the total photon energy = the electron energy plus the work function, or

$$E = hf = qW + \Phi \qquad \text{(Eqn. 2.6)}$$

Wavelength (nm)	Photon Energy (eV)	Electron Stopping Potential, W (Volts)	Work Function, Φ(eV)
435.8	2.85	1.25	1.6
546.1	2.27	0.7	1.6
632.8	1.96	0.4	1.6

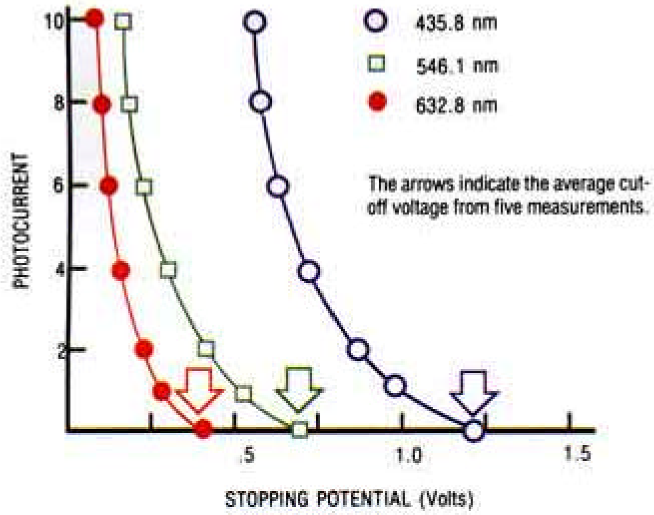


Figure 2.4 Results from typical demonstration of the photoelectric effect.

The brief introduction above does little justice to the diversity of processes that have produced these differing interpretations of light, and may leave the student confused. Is light a wave or a particle? Sadly, the correct answer is "yes". The interpretation to be used really depends upon the process being observed (the experiment being done). Often, the correct perspective depends upon the energies of the photons (frequencies). It is generally found that the wave aspects dominate at frequencies below about 10^{15} Hz and the particle aspects at higher frequencies. In the visible part of the spectrum both descriptions are useful.

Illustration of images at varying wavelength (energy).

As a conclusion to this segment, an unusual view of the electromagnetic spectrum is given by this illustration of the milky way galaxy as observed by a variety of instruments.

1) Radio Continuum (408 MHz) - Intensities: 10-4250 K

Intensity of radio continuum emission from hot, ionized gas in the Milky Way, from surveys with ground-based radio telescopes (Jodrell Bank MkI and MkIA, Bonn 100 meter, and Parkes 64 meter). At this frequency, most of the emission is from the scattering of free electrons in interstellar plasmas. Near some discrete sources, such as the supernova remnant Cas A near 110 degrees longitude, a significant fraction of the emission also comes from electrons accelerated in strong magnetic fields. The emission from Cas A is so intense that the diffraction pattern of the support legs for the radio receiver on the telescope is visible as a 'cross' shape.

Reference: Haslam, C. G. T., Salter, C. J., Stoffel, H., & Wi lson, W. E. 1982, Astron. Astrophys. Suppl. Ser., 47, 1

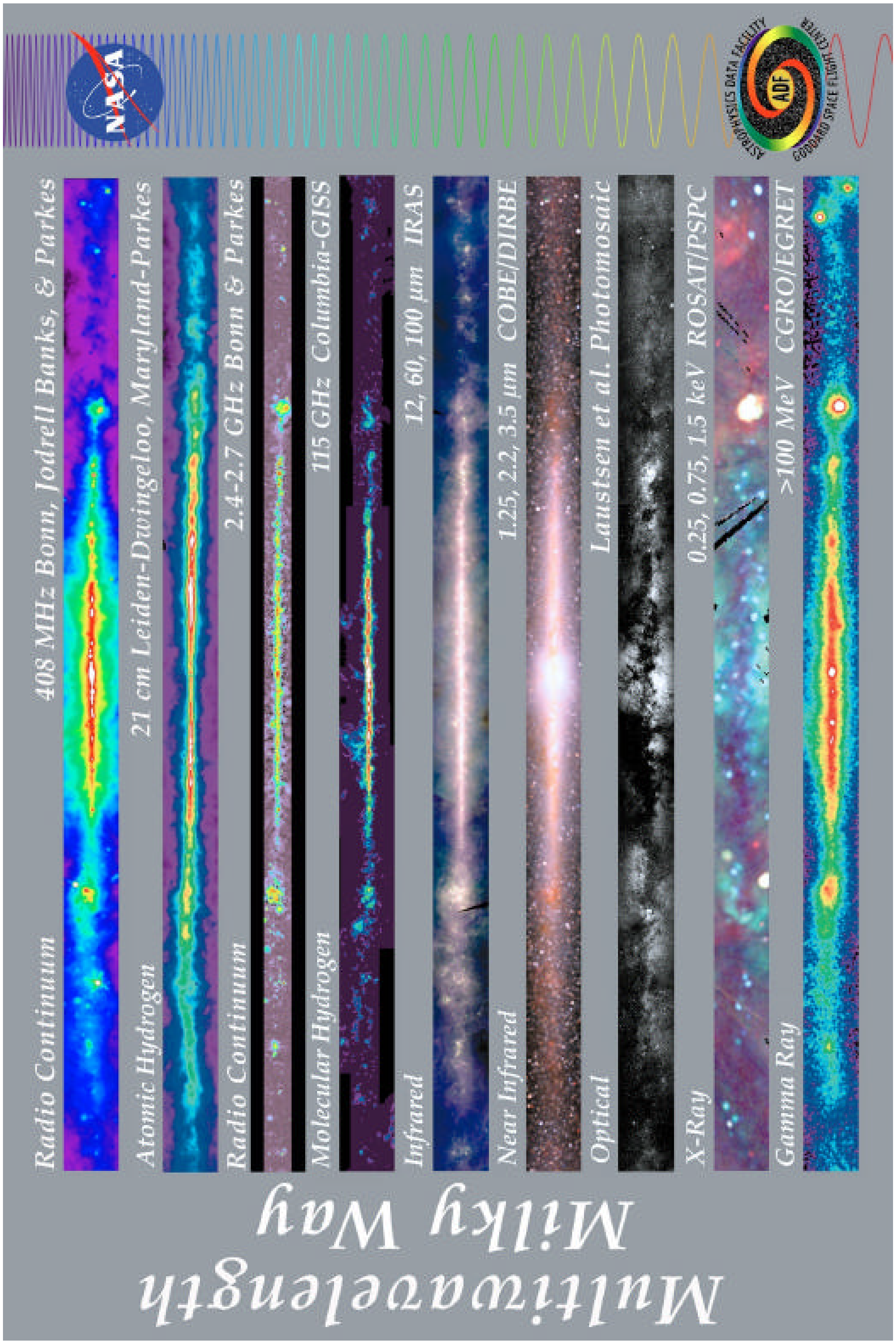


Figure 2.5 Multiwavelength Milky Way http://adc.gsfc.nasa.gov/mw/milkyway.html

2) Atomic Hydrogen - 1.4 GHz - Column densities: 10×10^{20} -230×10^{20} cm^{-2}

Column density of atomic hydrogen, derived on the assumption of optically thin emission, from radio surveys of the 21-cm spectral line of hydrogen. On a large scale the 21-cm emission traces the "warm" interstellar medium, which is organized into diffuse clouds of gas and dust that have sizes of up to hundreds of light years. Most of the image is based on the Leiden-Dwingeloo Survey of Galactic Neutral Hydrogen, made available by the authors in advance of publication. This survey was conducted over a period of 4 years using the Dwingeloo 25-m radio telescope, operated by the Netherlands Foundation for Research in Astronomy. The data were corrected for sidelobe contamination in collaboration with the University of Bonn.

References: Burton, W. B. 1985, Astron. Astrophys. Suppl. Ser., 62, 365 ; Hartmann, Dap, & Burton, W. B., "Atlas of Galactic Neutral Hydrogen," Cambridge Univ. Press, (1997, book and CD-ROM); Kerr, F. J., et al. 1986, Astron. Astrophys. Suppl. Ser.

3) Radio Continuum (2.4-2.7 GHz)

Intensity of radio continuum emission from hot, ionized gas and high-energy electrons in the Milky Way, from surveys with both the Bonn 100 meter, and Parkes 64 meter radio telescopes. Unlike the other views of our Galaxy presented here, these data extend to latitudes of only 5° from the Galactic midplane. The majority of the bright emission seen in the image is from hot, ionized regions, or is produced by energetic electrons moving in magnetic fields. The higher resolution of this image, relative to the 408 MHz picture above, shows Galactic objects in more detail. Note that the bright "ridge" of Galactic radio emission, appearing prominently in the 408 MHz image, has been subtracted here in order to show Galactic features and objects more clearly.

References:
Duncan, A. R., Stewart, R. T., Haynes, R. F., & Jones, K. L. 1995, Mon. Not. Roy. Astr. Soc., 277, 36
Fuerst, E., Reich, W., Reich, P., & Reif, K. 1990, Astron. Astrophys. Suppl. Ser., 85, 691
Reich, W., Fuerst, E., Reich, P., & Reif, K. 1990, Astron. Astrophys. Suppl. Ser., 85, 633

4) Molecular Hydrogen - Frequency: 115 GHz

Column density of molecular hydrogen inferred from the intensity of the J = 1-0 spectral line of carbon monoxide, a standard tracer of the cold, dense parts of the interstellar medium. Such gas is concentrated in the spiral arms in discrete "molecular clouds" and most molecular clouds are sites of star formation. The molecular gas is predominantly H2, but H2 is difficult to detect directly at interstellar conditions and CO, the second most abundant interstellar molecule, is observed as a surrogate. The column densities were derived on the assumption of a constant proportionality between the column density of H2 and the intensity of the CO emission. Black areas in the image indicate regions not yet surveyed for CO.

References: Dame, T. M., et al. 1987, Astrophysical Journal, 322, 706; Digel, S. W., & Dame, T. M. 1995, unpublished update

5) Infrared - Frequencies: 3.0×10^{3}-25×10^{3} GHz

Intensities: 0.25-100 (12 microns), 1.5-750 (60 microns), 12-750 MJy sr^{-1} (100 microns)
Composite mid and far-infrared intensity observed by the Infrared Astronomical Satellite (IRAS) in 12, 60, and 100 micron wavelength bands. The images are encoded in the blue, green, and red color ranges, respectively. Most of the emission is thermal, from interstellar dust warmed by absorbed starlight, including that in star-forming regions embedded in interstellar clouds. The image here is a mosaic of IRAS Sky Survey Atlas plates; emission from interplanetary dust in the solar system, the "zodiacal emission," was modeled and subtracted in the production of the Atlas at the Infrared Processing and Analysis Center (IPAC). The black, wedge-shaped areas indicate gaps in the IRAS survey.

Reference: Wheelock, S. L., et al. 1994, IRAS Sky Survey Atlas Explanatory Supplement, JPL Publication 94-11 (Pasadena: JPL) Order: CASI HC A08/MF A02

6) Near Infrared - Frequencies: 86 x 10^3-240 x 10^3 GHz

Intensities: 0.5-9 (1.25 microns), 0.35-20 (2.2 microns), 0.22-4.5 MJy sr^{-1} (3.5 microns)
Composite near-infrared intensity observed by the Diffuse Infrared Background Experiment (DIRBE) instrument on the Cosmic Background Explorer (COBE) in the 1.25, 2.2, and 3.5 micron wavelength bands. The images are encoded in the blue, green, and red color ranges, respectively. Most of the emission at these wavelengths is from cool, low-mass K stars in the disk and bulge of the Milky Way. Interstellar dust does not strongly obscure emission at these wavelengths; the maps trace emission all the way through the Galaxy, although absorption in the 1.25 micron band is evident in the general direction of the Galactic center.
Reference: Hauser, M. G., Kelsall, T., Leisawitz, D., & Weiland, J. 1995, COBE Diffuse Infrared Background Experiment Explanatory Supplement, Version 2.0, COBE Ref. Pub. No. 95-A (Greenbelt, MD: NASA/GSFC)

7) Optical - Frequency: 460 x 10^3 GHz

Intensity of visible light from a mosaic of wide-field photographs by Laustsen, Madsen, & West (1987). Scanned images of the original prints were kindly provided by C. Madsen (European Southern Observatory, cmadsen@eso.org). The images are copyright (1987) by ESO. Owing to the strong obscuration by interstellar dust the light is primarily from stars within a few thousand light-years of the Sun, nearby on the scale of the Milky Way, which has a diameter on the order of 100,000 light years. Nebulosity from hot, low-density gas is widespread in the image. Dark patches are due to absorbing dust clouds, which are evident in the Molecular Hydrogen and Infrared maps as emission regions. The mosaic is constructed from eight photographs. Narrow, vertical gaps are evident between some photographs, as are slight discontinuities in brightness.
Reference: Laustsen, S., Madsen, C., West, R. 1987, Exploring the Southern Sky, (Berlin: Springer-Verlag)

8) X-Ray - Frequency: 60-360 x 10^6 GHz

Intensities: 0-20 (0.25 keV), 0-10 (0.75 keV), 0-10 x 10^{-4} photons $arcmin^{-2}$ s^{-1} (1.5 keV)
Composite X-ray intensity observed by the Position-Sensitive Proportional Counter (PSPC) instrument on the Röntgen Satellite (ROSAT). Images in three broad, soft X-ray bands centered at 0.25, 0.75, and 1.5 keV are encoded in the red, green, and blue color ranges, respectively. In the Milky Way, extended soft X-ray emission is detected from hot, shocked gas. At the lower energies especially, the interstellar medium strongly absorbs X-rays, and cold clouds of interstellar gas are seen as shadows against background X-ray emission. Color variations indicate variations of absorption or of the temperatures of emitting regions. The black regions indicate gaps in the ROSAT survey.
Reference: Snowden, S. L., et al. 1995 Astrophys. J., 454, 643

9) Gamma Ray - Frequencies: >2.4 x 10^{13} GHz

Intensities: 4 x 10^{-5} - 93 x 10^{-5} photons cm^{-2} s^{-1} sr^{-1}
Intensity of high-energy gamma-ray emission observed by the Energetic Gamma-Ray Experiment Telescope (EGRET) instrument on the Compton Gamma-Ray Observatory (CGRO). The image includes all photons with energies greater than 100 MeV. At these extreme energies, most of the celestial gamma rays originate in collisions of cosmic rays with hydrogen nuclei in interstellar clouds. The bright, compact sources near Galactic longitudes 185°, 195°, and 265° indicate high-energy phenomena associated with the Crab, Geminga, and Vela pulsars, respectively.
References: Hunter, S. D., et al. 1997, Astrophys. J., 481, 205; Thompson, D. J., et al. 1996, Astrophys. J. Suppl., 107, 227

B Polarization of radiation

The discussion above treats electric and magnetic fields as scalars - that is, amplitudes only. The alert students probably noticed at the beginning of this section that $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are both vectors. This vector character to EM radiation becomes important when we consider the concept of polarization. Familiar to most as an aspect of expensive sunglasses, polarization shows up in visible light observations and radar. A brief illustration of how EM waves propagate becomes necessary at this point.

Figure 2.6 shows how the electric and magnetic fields oscillate with respect to one another in an EM wave (in vacuum).

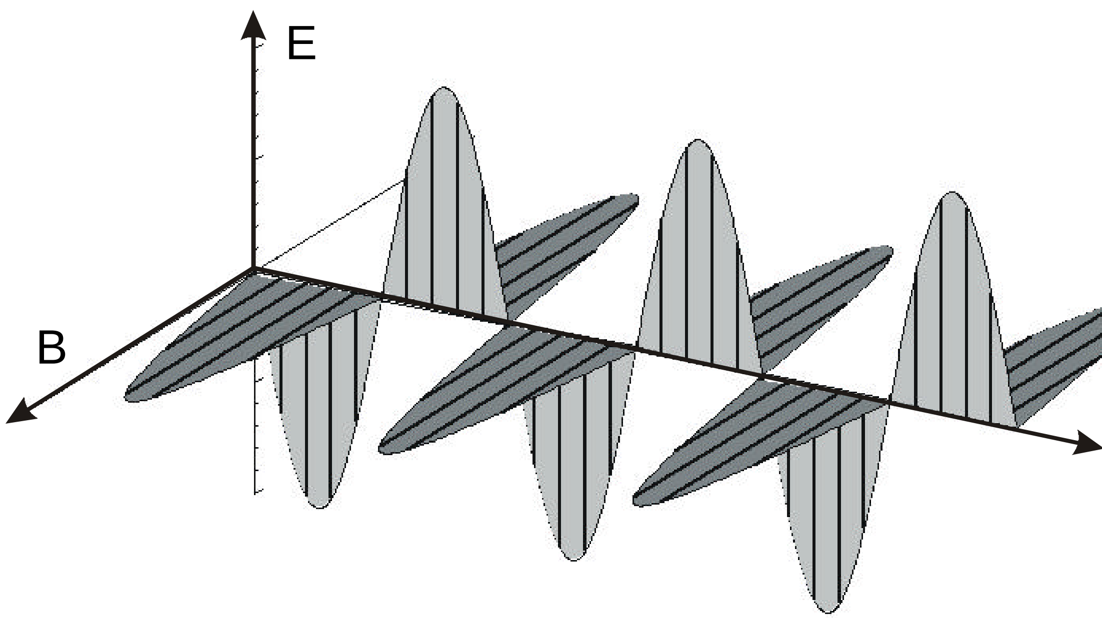


Figure 2.6 An electromagnetic wave. Note E⊥B, and both are perpendicular to the direction of propagation. Following a typical convention, E is in the x direction, B is in the y direction, and the wave is propagating in the z direction.

Other forms of polarization are possible, but harder to illustrate.

C Sources of Electromagnetic Radiation

There are several major sources of electromagnetic radiation, all ultimately associated in some for with the acceleration (change of energy) of charged particles (mostly electrons). For purposes of remote sensing, these can be distinguished into three categories:

(1) Individual atoms or molecules which radiate line spectra
(2) Hot, dense bodies which radiate a continuous "black-body" spectrum
(3) Electric currents moving in wires (aka antenna)

The first two of these sources of radiation are now considered in somewhat more detail.

1 Line spectra

Single atoms or molecules emit light in a form termed "line spectra". An atom or molecule, which is reasonably isolated (such as in a gas at ordinary temperatures and pressures) will **radiate** a discrete set of frequencies called a line spectrum. If, on the other had, we pass radiation having a continuous spectrum of frequencies through a gas we find that a discrete set of frequencies is **absorbed** by the gas leading to a spectrum of discrete absorption lines

The wavelengths radiated (absorbed) are characteristic of the particular atom or molecule and thus represent a powerful tool for determining the composition of radiating (or absorbing) gases. Much of our knowledge of the chemical composition of stars (including our sun) comes from detailed analysis of such line spectra.

The process is reasonably well explained by the Bohr model of the atom, as developed at the beginning of this century. The model, developed analytically in an appendix, uses the somewhat familiar construct of the atom as a small solar system, with a nucleus at the center composed of the heavy protons (+) and neutrons. The lighter electrons (-) orbit the nucleus at well-defined radii, which correspond to different energy levels. The closer they orbit to the nucleus, the lower (more negative) their energy levels are. As energy is given to the electrons, the radii of their orbits increase, until they finally break free. Bohr hypothesized that the radii of the orbits were constrained by quantum mechanics to have certain values (really, a constraint on angular momentum). This produces a set of discrete energy levels that are allowed for the electrons. Bohr then said that the emission and absorption of energy (light) by an atom could only occur for transitions between the discrete energy levels allowed to the electrons.

A few pages of mathematics (see appendix) gives the formula for the energy of the electrons orbiting in hydrogen-like atoms:

$$E = -\frac{1}{2}\left(\frac{Z e^2}{4\pi \varepsilon_0 \hbar}\right)^2 \frac{m}{n^2} = Z^2 \frac{E_1}{n^2} \qquad \text{(Eqn. 2.7)}$$

where $$E_1 = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2} = -13.58\text{ eV}$$

(here, n = the quantum number: 1, 2, 3, ….; Z - the atomic number;
m = the electron mass; e= the electron charge; the remaining terms are constants.)

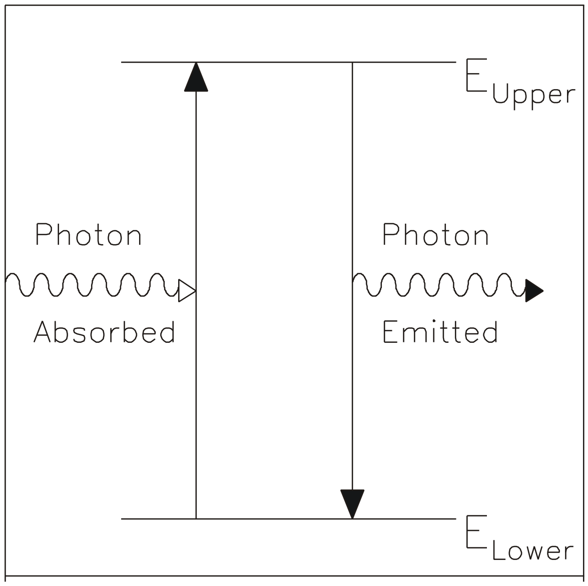


Figure 2.7 Bohr Postulate - photons produced/destroyed by discrete transition in energy.

Figure 2.8 illustrates the energy levels for the Bohr model of the hydrogen atom. We find that the ionization energy, the energy necessary to remove the electron from its "well", is 13.58 eV. The concept of a "work function", described above for the process of photoemission, derives from the same physics.

Within an atom, if the electron gains somewhat less energy, it may move up to an "excited state", where n > 1. For example, if an electron beginning in the ground state gains 10.19 eV it will move up to the n = 2 level. Dropping down from n = 2 to n = 1, it will emit a photon of 10.19 eV energy, at a wavelength:

$$\lambda = \frac{hc}{\Delta E} = 121.6\text{ nm} = 1216\text{ Å}$$

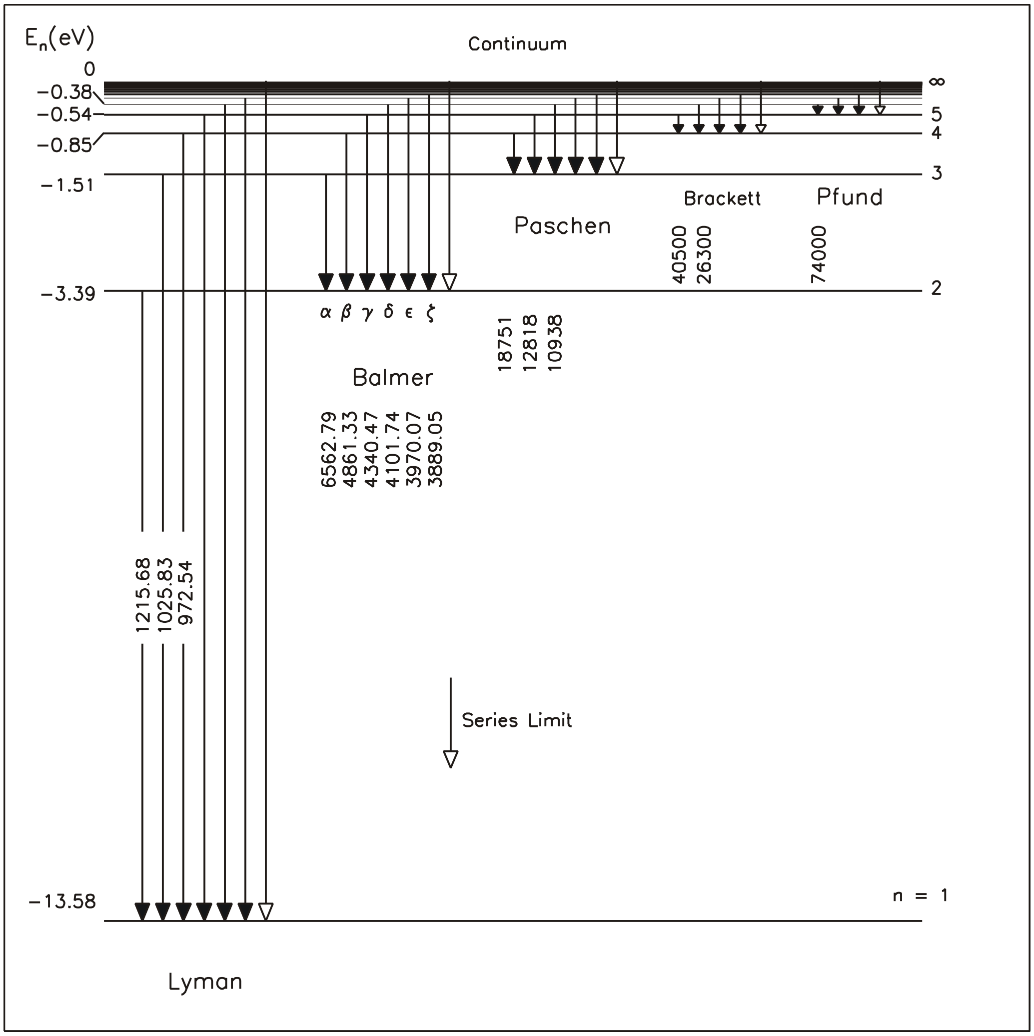


Figure 2.8. The energy level diagram of the hydrogen atom, showing the possible transitions corresponding to the different series. The numbers along the transitions are wavelengths. (Wavelengths are in units of Angstroms, where 1 nm = 10 A)
Adapted from: Fundamentals of Atomic Physics, Atam P. Arya, p264, 1971.

If ΔE is expressed in electron-volts (eV), which it usually is, then the constant "hc" in the numerator can be written as:

$$hc = 4.14\times10^{-15} \bullet 3\times10^{8} = 1.24\times10^{-6} \text{ (eV m)}$$

and thus the wavelength λ (in meters) is given by:

$$l\text{ (m)} = \frac{1.24\times10^{-6}}{\Delta E\text{ (eV)}} \quad \text{or} \quad l\text{ (nm)} = \frac{1240}{\Delta E\text{ (eV)}} \qquad \text{(Eqn. 2.8)}$$

In general, transitions will occur between different energy levels, resulting in a wide spectrum of discrete spectral lines. Transitions from (or to) the n = 1 energy level (the ground state) are called the Lyman series. The n = 2 to n = 1 transitions is the Lyman alpha (α) transition. This ultraviolet (UV) emission is one of the primary spectral (emission) lines of the sun's upper atmosphere. The emission (or absorption) lines in the visible portion of the sun's spectrum are the Balmer series, transitions from $n > 2$ to $n = 2$. Higher order series are of less importance for our purposes.

The Bohr model is successful in predicting the observed energy levels for one-electron atoms. It is useful for illustrating the quantum nature of the atom, and the associated energy levels. It is also a good beginning for understanding all of the interesting spectral characteristics reflected and radiated light may exhibit in remote sensing applications. The Bohr model was ultimately replaced by the solution of the Schrödinger equation, and a more general form of quantum mechanics.

2 Black body radiation

Black Body Radiation is emitted by hot solids, liquids or dense gases and has a continuous distribution of radiated wavelength as shown in Figure 2.9. The curves in this figure give the radiance, L, in dimensions of $\frac{\text{Power}}{\text{unit area - wavelength - solid angle}}$, or units of $\frac{\text{Watts}}{\text{m}^2\ \mu\ \text{ster}}$. The radiance equation is:

$$Radiance = L = \frac{2\,\text{hc}^2}{\lambda^5}\frac{1}{e^{\frac{\text{hc}}{\lambda\,\text{kT}}}-1} \qquad \text{(Eqn. 2.9)}$$

where $\text{c} = 3\times10^8\ \frac{m}{s}$; $\text{h} = 6.626\times10^{-34}\ joule-s$; $\text{k} = 1.38\times10^{-23}\ \frac{Joule}{Kelvin}$

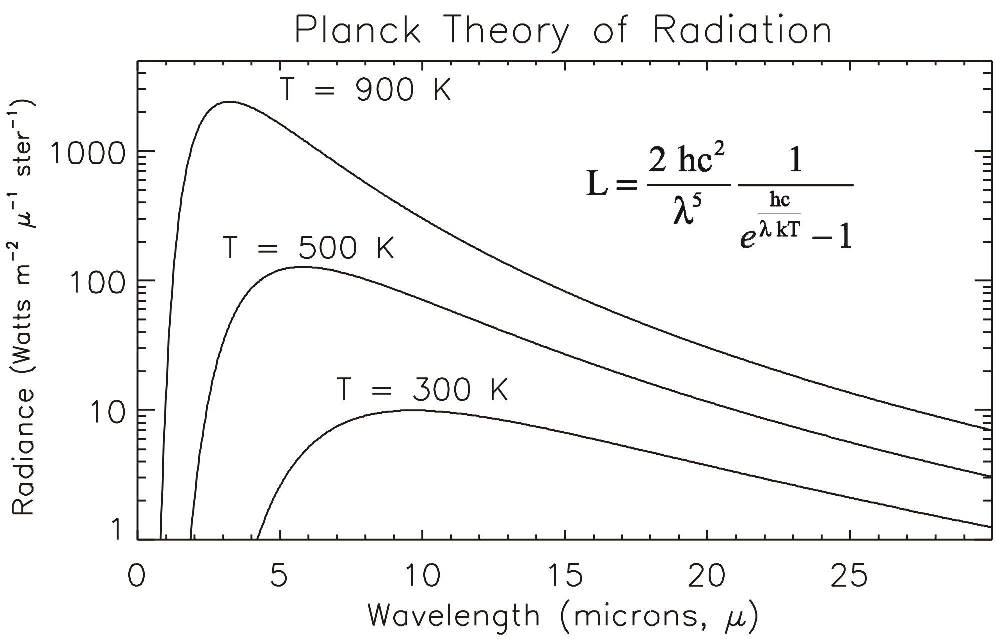


Figure 2.9 Blackbody radiation as a function of wavelength.

It is a little easier to decipher the nature of the formula if it is rewritten slightly:

$$\text{L} = \frac{2}{\text{c}^3\text{h}^4}\frac{\left(\frac{\text{hc}}{\lambda\,\text{kT}}\right)^5}{e^{\frac{\text{hc}}{\lambda\,\text{kT}}}-1}\left(\text{kT}\right)^5 \quad = \frac{2}{\text{c}^3\text{h}^4}\frac{x^5}{e^x-1}\left(\text{kT}\right)^5 \qquad \text{(Eqn. 2.10)}$$

where the dimensionless term: $x = \left(\frac{\text{hc}}{\lambda\,\text{kT}}\right)$ is defined. We see that the shape of this function of wavelength (λ) is unchanging as temperature changes - only the overall amplitude changes (and of course, the location of the peak in wavelength).

Real materials will differ from the idealized black body in their emission of radiation. The emissivity of a surface is a measure of the efficiency with which the surface absorbs (or radiates) energy and lies between 0 (for a perfect reflector) and 1 (for a perfect absorber). A body which has $\varepsilon = 1$ is called a black body. In the infrared, many objects are nearly black bodies, in particular vegetation. Materials with $\varepsilon<1$ are called gray bodies. Note that the emissivity (ε) will vary with wavelength.

Note that in some textbooks, a slightly different form of Planck's Law may be found, where an extra π is included:

$$\text{Radiant Exitance} = M = \frac{2\,\pi\, hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda\, kT}} - 1} \quad \frac{\text{Watts}}{m^2\, \mu} \qquad \text{(Eqn. 2.11)}$$

The difference is that the dependence on angle of the emitted radiation has been removed by integrating over the solid angle. You can do this for black-bodies because they are 'Lambertian' surfaces by definition - the emitted radiation does not depend upon angle, and $M = \pi L$.

For our purposes we are particularly interested in two aspects of the Planck curves:

a) the total power radiated which is represented by the area under the curve, and
b) the wavelength at which the curve peaks, λ_{max}.

The power radiated (integrated over all wavelengths) is given by

$$R = \sigma\, \varepsilon\, T^4 \left(\text{Watts}/m^2 \right) \quad \text{(Stefan Boltzmann Law)} \qquad \text{(Eqn. 2.12)}$$

where R = Power radiated / m^2

ε = Emissivity (taken as unity for black body)
$\sigma = 5.67 \times 10^{-8} \left(W/m^2\, K^4 \right)$ (Stefan's Constant)
T = Temperature of the radiator (in K)

Wien's Displacement Law gives the wavelength at which the peak in radiation occurs:

$$l_{max} = a/T \qquad \text{(Eqn. 2.13)}$$

for a given temperature T. The constant "a" has the value

$$a = 2.898 \times 10^{-3} \, [\text{m K}]$$

which gives λ_{max} in meters if T is K.

Example:

Assume that the sun radiates like a blackbody, which is not a bad assumption, though we must choose two slightly different temperatures to match the observed quantities.

(a) Find the wavelength at which this radiation peaks, $\lambda_{max.}$ The solar spectral shape in the visible is best matched by a temperature of ~ 6000 K.
(b) Find the total power radiated by the sun. The Stefan-Boltzmann law is best served by an "effective temperature" of ~ 5800 K.

$$\text{(A)} \qquad \lambda_{max} = \frac{a}{T} = \frac{2.898 \times 10^{-3}\ (m/K)}{6000\ K} = 4.83 \times 10^{-7}\ m$$

The spectrum peaks at ~500 nm, as illustrated below in figure 2.10.

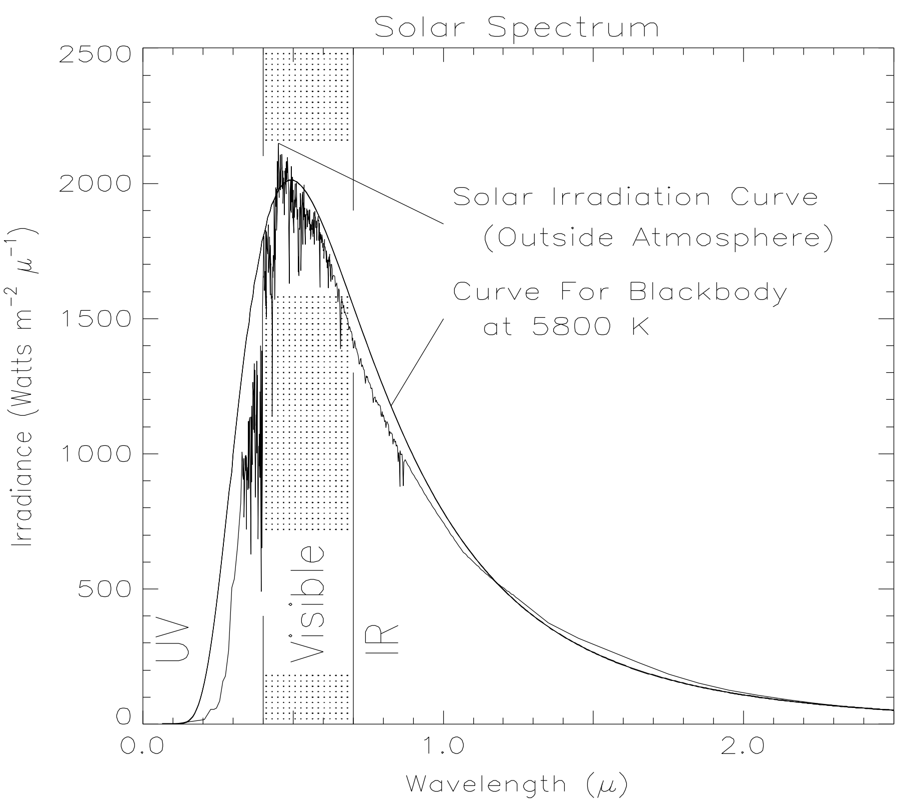


Figure 2.10 - The solar spectrum, based on the spectrum of Neckel and Labs “The solar radiation between 3300 and 12500 Angstrom", Solar Physics, 90, 205-258, 1984. Data file courtesy of Bo-Cai Gao, NRL. The peak occurs at about 460 nm (blue).

(B) Next we can calculate R, the power emitted per-square-meter of surface. We use:

$$R = \sigma \varepsilon T^4$$

and we assume that $\varepsilon = 1$ (Black Body). Evaluating, we get:

$$R = 5.67 \times 10^{-8} \bullet 1 \bullet 5800^4 = 6.42 \times 10^7 \frac{\text{Watts}}{\text{meter}^2}$$

To find the total solar power output we must multiply by the solar surface area, $S_\odot = 4 \pi R_\odot^2$, where $R_\odot = 6.96 \times 10^8$ m is the mean radius of the sun. Hence the total solar power output is:

$$P_\odot = R \, (4 \pi R_\odot^2)$$

$$= 4 \pi (6.96 \times 10^8)^2 \times (6.42 \times 10^7)$$

$$P_\odot = 3.91 \times 10^{26} \ \text{W}$$

(Not bad for a little star.)(See Kenneth Phillips, Guide to the Sun, Cambridge Press, 1992, pages 83-84) The sun's spectrum is shown in figure 2.10, with the spectrum of a 5,800 K black body superimposed.

A slightly different perspective on the visible portion of the solar spectrum is obtained from an illustration created at the University of Hawaii. The dark lines superimposed on the rainbow scale are the solar absorption features, also known as Frauenhofer lines. These are due to the cool hydrogen and helium and other elements just above the surface of the sun. These correspond to the dips in the solar irradiation curve illustrated above.

Figure 2.11 - Solar spectrum as observed on Mount Haleakala, courtesy of the University of Hawai`i, Institute for Astronomy, C.E.K. Mees Solar Observatory, http://www.solar.ifa.hawaii.edu/mees.html

D EM Radiation matter interactions

(This section from Avery and Berlin)

Electromagnetic radiation manifests itself only through its interactions with matter, which can be in the form of a gas, a liquid, or solid. This concept is clearly illustrated by shining a flashlight beam of visible light on a white wall in a darkened room. If we stand at a right angle to the long axis of the beam, the light is visible only at its source **E** and where it strikes the wall and is reflected to our eyes. The beam cannot be seen from the side and can be made visible only when its optical path contains particles large enough to scatter some of the light beam sideways. This can be accomplished by adding chalk dust or smoke to the invisible beam. Their large particles will scatter a portion of the EMR to our eyes, enabling the beam to be seen from the side. This side scattering of visible light along a beam path is known as the **Tyndall effect.**

EMR that impinges upon matter is called **incident radiation.** For the earth, the strongest source of incident radiation is the sun. Such radiation is called **insolation** a shortening of incoming solar radiation. The full moon is the second strongest source, but its radiant energy measures only about one millionth of that from the sun. When EMR strikes matter, EMR may be **transmitted, reflected, scattered,** or **absorbed** in proportions that depend upon:

1) the compositional and physical properties of the medium,
2) the wave- length or frequency of the incident radiation, and
3) the angle at which the incident radiation strikes a surface.

The four fundamental energy interactions with matter are illustrated in Figure 2-12.

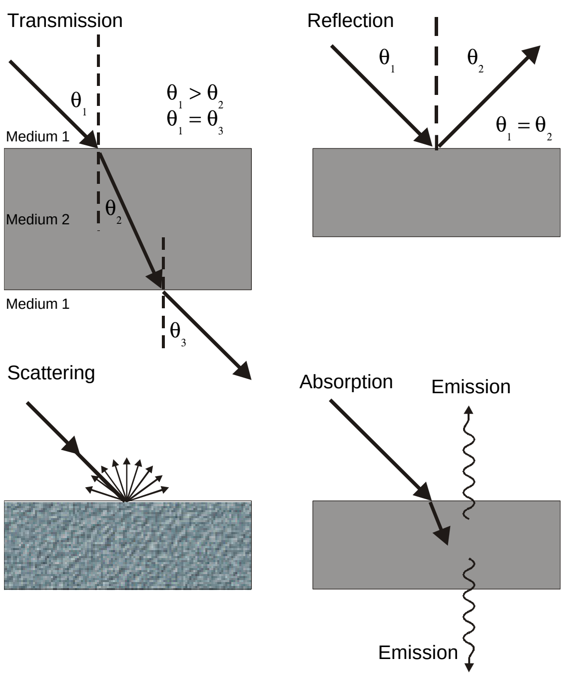


Figure 2-12
Citation: Thomas Eugene Avery and Graydon Lennis Berlin, Fundamentals of Remote Sensing and Airphoto Interpretation, Macmillan Publishing Company, NY, NY, 10022, 1992

1 Transmission

Transmission is the process by which incident radiation passes through matter without measurable attenuation; the substance is thus transparent to the radiation. Transmission through material media of different densities (e.g., air to water) causes the radiation to be refracted or deflected from a straight-line path with an accompanying change in its velocity and wavelength; frequency always remains constant. In Figure 2-10, it is observed that the incident beam of EMR (θ_1) is deflected toward the normal in going from a medium of low density to a denser medium (θ_2). Upon emerging from the other side of the denser medium, the beam is refracted away from the normal (θ_3). The angle relationships in Figure 2-9 are $\theta_1 > \theta_2$ and $\theta_1 = \theta_3$.

The change in EMR velocity is explained by the **index of refraction** (n), which is the ratio between the velocity of EMR in a vacuum (c) and its velocity in a material medium (v).

$$n = \frac{c}{v} \qquad \text{(Eqn. 2.14)}$$

The index of refraction for a vacuum (perfectly transparent medium) is equal to 1, or unity. Because *v* is never greater than c, *n* can never be less than 1 for any substance. Indices of refraction vary from 1.0002926 for the earth's atmosphere, to *1.33* for water, to 2.42 for a diamond. The above relation leads to Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \text{(Eqn. 2.15)}$$

2 Reflection

Reflection (also called specular reflection) describes the process whereby incident radiation "bounces off" the surface of a substance in a single, predictable direction. The angle of reflection is always equal and opposite to the angle of incidence ($\theta_1 = \theta_2$ in Figure 2-10). Reflection is caused by surfaces that are smooth relative to the wavelengths of incident radiation. These smooth, mirror-like surfaces are called specular reflectors**.** Specular reflection causes no change to either EMR velocity or wavelength.

3 Scattering

Scattering (also called diffuse reflection) occurs when incident radiation is dispersed or spread out unpredictably in many different directions, including the direction from which it originated (Figure 2-10). In the real world, scattering is much more common than reflection. The scattering process occurs with surfaces that are rough relative to the wavelengths of incident radiation. Such surfaces are called diffuse reflectors. EMR velocity and wavelength are not affected by the scattering process.

4 Absorption

Absorption is the process by which incident radiation is taken in by the medium (Figure 2-10). For this to occur, the substance must be opaque to the incident radiation. A portion of the absorbed radiation is converted into internal heat energy, which is subsequently **emitted** or **reradiated** at longer thermal infrared wavelengths (Figure 2-10).

5 Energy Balance

The interrelationships between energy interactions, as a function of wavelength (λ), can be expressed in the following manner:

$$E_I(\lambda) = E_T(\lambda) + E_R(\lambda) + E_A(\lambda)$$

where: $E_I(\lambda) =$ incident radiant energy,

$E_T(\lambda) =$ fraction transmitted,

$E_R(\lambda) =$ fraction reflected (specular and diffuse) , and

$E_A(\lambda) =$ fraction absorbed.

Most opaque materials transmit no incident radiant energy; hence, E_T (λ) = 0, and $E_R(\lambda) + E_A(\lambda) = 1 = E_I(\lambda)$. In regard to visible light,

1) clear glass would have a high transmission value and low reflection and absorption values;
2) fresh snow would have a high reflectance value and low transmission and absorption values; and
3) fresh asphalt would be characterized by a high absorption value and minimal transmission and reflection values.

Because only the part of incident radiation that is absorbed by an object is effective in heating it, there would only be a minuscule rise in temperature for glass and snow, whereas the asphalt's temperature would be markedly higher.

E Problems:

1. MWIR radiation covers the 3-5μ portion of the EM spectrum. What energy range does this correspond to, in eV.

2. What frequency is an x-band radar? What wavelength?

3. What is the ground state energy for an He+ ion, in eV. (Note, Z=2)

4. Calculate the energy (in eV), frequency (in Hz), and wavelength (in meters, microns, and nano-meters) for the n=4 to n=2 transition in a hydrogen atom (note, Z=1). This is the Balmer-β transition.

5. Student exercise: check long wavelength behavior - use the fact that for small x: $e^x - 1 \approx x$ to get rid of the exponential term in the denominator. Also, which term dominates L for small wavelength?

6. Calculate the radiance, L(λ), for T = 1000 K, from λ=0-20μ, and plot. Note that this is an exercise in calculation, and you should be sure you can obtain the correct answer with a hand calculator at a minimum of 2 wavelengths - say 3 and 10 μ.

7. Calculate the peak wavelength for radiation at T=297 K, 1000 K, and 5800 K, in microns and nano-meters.

8. Calculate the radiated power for a black body at 297 K, in Watts/m^2.

9. Calculate the radiated power for a gray body at 297 K, ε=0.8, in Watts/m^2. Assume a surface area of 2 m^2, and calculate the radiated power, in Watts

10. Snell's law in optics is normally given as:

$$n_1 \sin q_1 = n_2 \sin q_2$$

Where the angles are defined as in Figure 2.10. For an air/water interface, one can use the typical values: $n_1 = 1$; $n_2 = 1.33$. For such values, calculate q_2 if $q_1 = 30^\circ$. What is the speed of light in water?

1 Black Body Problem

Calculate the equilibrium temperature of the earth, or any satellite orbiting the sun.

a How much energy is radiated by the sun, and how much then reaches earth orbit?

($R_{sun} = 6.96\times10^{8}$ m; $T_{sun} = 5800$ K; $\sigma = 5.67\times10^{-8}\ \mathrm{W/m^2K^4}$)

The radiated power is: $P_{sun} = \varepsilon\ \sigma\ T_{sun}^4 \bullet 4\pi R_{Sun}^2 =$

Textbook Answer: 3.83×10^{26} Joules/s

b The radiation spreads as the electromagnetic energy moves outwards, according to Gauss' Law (conservation of energy, really). What is the radiation at earth's orbit?

$\left(R_{\text{Earth Orbit}} = 150\times10^{9}\ \text{m, aka 93 million miles}\right)$

$R = P_{solar} \big/ \left(4\pi\ R_{\text{Earth Orbit}}^2\right) =$

The textbook value is: 1381 $\mathrm{Watts/m^2}$

c How much power does the earth receive?

The radiation is incident on a cross-section area which is the area of the earth's disk: $\pi\ R_{Earth}^2$. $\left(R_{Earth} = 6380\ \text{km}\right)$. One question which can be left open at this point is the question of the earth's albedo - the factor which determines what fraction of the sunlight incident on the earth is absorbed, vs reflected. Here, we will use the symbol "α" for albedo.

$$P_{Solar} = \alpha\ \sigma\ T_{Sun}^4 \bullet \left(\frac{R_{Sun}}{R_{\text{Earth Orbit}}}\right)^2 \bullet \pi\ R_{Earth}^2 = \text{Watts}$$

where you can nominally take α to be one for now.

d This power has to be re-radiated by the earth - once a steady state temperature has been reached. Hence, you can derive a temperature for the earth.

$$P_{Solar} = \alpha\ \sigma\ T_{Sun}^4 \bullet \left(\frac{R_{Sun}}{R_{\text{Earth Orbit}}}\right)^2 \bullet \pi\ R_{Earth}^2 = \varepsilon_{Earth}\ \sigma\ T_{Earth}^4 \bullet 4\pi\ R_{Earth}^2$$

$T_{Earth} = 280$ K

The earth will freeze if the number drops to 273 K. If the emissivity of the earth is one, what does α have to drop to for this to happen?